



The new Procedures for Diverge, Merge, and Small Weaving Segments in the new Version of German Highway Capacity Manual (HBS 2015)

Ning Wu¹ and Kerstin Lemke²

¹*Institute for Traffic Engineering and Management, Ruhr University Bochum, Germany*

²*Federal Highway Research Institute (BASt), Bergisch Gladbach, Germany
ning.wu@rub.de, Lemke@bast.de*

Abstract

The chapter for diverge, merge, and small weaving segments in the new German Highway Capacity Manual (HBS, edition 2015) is rewritten in accordance with forthcoming developments in the past 10 years. In this paper, an overview of the chapter in HBS 2015 is presented. Differences between the first edition 2001 and the new edition of the chapter in the German Highway Capacity Manual are indicated and discussed. In addition, the results of two cases which are not yet included in HBS 2015 are presented.

Keywords: Capacity, Merge, Diverge, Small Weaving Segment, Level of Service Concepts

1 Introduction

The first edition of German Highway Capacity Manual (HBS) was published in 2001 (FGSV, 2001). Now, a new edition was published in 2015 (FGSV, 2015). For the new German Highway Capacity Manual, most major chapters were revised and some of them were totally rewritten.

The joint chapter for diverge, merge, and small weaving segments (not longer than 500m) is a new development in accordance with forthcoming research works in the past 10 years. An overview of the chapter in HBS 2015 is presented in this paper. Procedures dealing with performance analyses and Level of Service (LOS) of those segments are introduced both for freeways and rural highways. Differences between the former HBS 2001 and HBS 2015 are indicated and discussed.

In most of the existing highway capacity manuals, LOS of diverge, merge, and small weaving segments is traditionally defined by speed, volume, or density in critical areas (FGSV, 2001; TRB, 2000, 2010). In that traditional concept, several capacity values of different critical areas (diverge, merge, and weaving) as well as upstream and downstream major-road segments within the influence areas are evaluated separately. In HBS 2015, a new model which considers the total diverge, merge, and weaving segment as an entire object is incorporated. A combined volume-to-capacity ratio (freeways) or a combined density (rural highways) is used for defining the LOS of the total segment.

The parameters of the new procedure are functions of the number of lanes of the major road, the number of lanes in the on-ramp or off-ramp, and the predefined geometric design of those segments. The model parameters are calibrated with field data or defined by experts' experiences within a set of coefficients. With those procedures, the traffic quality (LOS) can be obtained directly as a function of the volumes or densities on the major road and on the on-ramp or off-ramp respectively. The new procedure has the following advantages: a) a uniform function for all types of diverge, merge, and small weaving segments, b) traffic quality assessment for all critical areas under investigation in one step, and c) the procedure can easily be calibrated. For applications in practice, a set of monographs is provided.

The paper is organized as follows. In the following section 2, the chapter dealing with diverge, merge, and small weaving segments on freeways is presented. In this chapter, a short description of the embedded new model is given. In section 3, the chapter dealing with diverge, merge, and small weaving segments on rural highways is presented. In section 4, two additional cases for diverge and merge on 4 lanes carriageways are investigated according to additional data collected. Finally, a summary and outlook is given in the concluding section 5.

2 The Chapter for Diverge, Merge, and Small Waving Segments on Freeways in HBS 2015

In HBS 2015, LOS on freeways is defined according the volume-to-capacity ratio. This applies both for basic freeways segments and for diverge, merge, and small weaving segments. The definition of LOS is given in Table 1.

LOS	volume-to-capacity ratio $x [-]$
A	$\leq 0,30$
B	$\leq 0,55$
C	$\leq 0,75$
D	$\leq 0,90^{1)}$
E	$\leq 1,00$
F	$> 1,00$

¹⁾ 0,92 for on-ramp types E 1 and E 2 with ramp-metering

Table 1: Volume-to capacity thresholds for defining LOS of freeways in HBS 2015

2.1 The new LOS Model of Combined Volume-to-Capacity Ratio for Diverge, Merge, and Small Waving Segments on Freeways

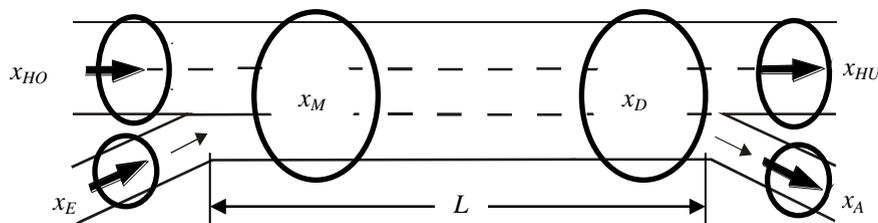


Figure 1: Volume-to-capacity ratios in different areas of on-ramp, off-ramp, and weaving segments

In the traditional concept of LOS, several capacity values of different critical areas (diverge, merge, and weaving) as well as upstream and downstream basic road segments within the influence areas are evaluated separately. In HBS 2015, a new model (Wu and Lemke, 2011) which considers the total diverge, merge, and weaving segment as an entire object is incorporated. For a better understanding of this paper, a short introduction of the model is given in the following.

In general, at a diverge or merge point the volume-to-capacity ratio has to be the same on both sides of the diverge or merge point (Wu, 1997). Thus, the following equation holds for the case that the traffic volumes are equally distributed on all freeway segments and ramps under consideration (cf. Figure 1):

$$x_E^{a1} + x_{HO}^{b1} = x_M^{c1} \quad \text{and} \quad x_D^{c2} = x_A^{a2} + x_{HU}^{b2} \quad (1)$$

- with $x_{XX} = q_{XX}/C_{XX}$
 $=$ volume-to-capacity ratio of the corresponding cross-section XX
 $q_{XX}, C_{XX} =$ volume and capacity of the corresponding cross-section XX , pc/h
 $HO, HU =$ index for the upstream and downstream major freeway
 $E, A =$ index for the on-ramp (entrance) and off-ramp (exit)
 $M, D =$ index for the total merge (or weaving) and diverge segment

The parameters $a1, b1, c1$, and $a2, b2, c2$ can be calibrated against field data.

Normally, the values of the model parameters a, b, c are different and they are dependent on geometrical configurations of the corresponding legs (major freeway, on-ramp, or off-ramp) of the segment under investigation. As an approximation and simplification, $a = b = c$ is introduced. This simplification is not critical for practical applications because the resulting deviations for the capacities of the diverge and merge area are quite small. Thus,

$$x_M^a = x_E^a + x_{HO}^a \quad \text{and} \quad x_D^a = x_A^a + x_{HU}^a \quad (2)$$

At capacity with $x_M = 1$ and $x_D = 1$, is

$$x_M^a = x_E^a + x_{HO}^a = 1 \quad \text{or} \quad x_E^a = \sqrt[a]{1 - x_{HO}^a} \quad \text{and} \quad x_D^a = x_A^a + x_{HU}^a = 1 \quad \text{or} \quad x_A^a = \sqrt[a]{1 - x_{HU}^a} \quad (3)$$

With $x = q/C$, this corresponds to (cf. Figure 5, left)

$$q_{E,max} = C_E \cdot \sqrt[a]{1 - \left(\frac{q_{HO}}{C_{HO}}\right)^a} \quad \text{and} \quad q_{A,max} = C_A \cdot \sqrt[a]{1 - \left(\frac{q_{HU}}{C_{HU}}\right)^a} \quad (4)$$

The parameters C_A, C_E, C_{HO} , and C_{HU} are the capacities of the off-ramp, the on-ramp, the upstream major freeway, and the downstream major freeway.

Varying the model parameter a , the function can be fitted to the existing lineal capacity models (e.g. Brilon, Westphal, 1994; FGSV, 2001) in the diverge or merge area for different types of weaving segments (see Figure 5, left, an example for an on-ramp). As can be seen, the total capacity of on-ramps and small weaving segments, $q_{E,max}$ (or $q_{A,max}$ for off-ramps) are limited by the capacities C_E (or C_A for off-ramps) and C_{HO} (or C_{HU} for off-ramps) and the capacities according to the lineal models in the merge area C_M^* (or capacities in the diverge area C_D^* for off-ramps). Normally, the value of the model parameter a depends on the number of lanes on major freeways and the number of lanes on on-ramps (or off-ramps). For a small weaving segment, the value of the model parameter a depends also on the volume q_A in the following off-ramp. In addition, the model parameter a depends on the length L of the acceleration (or deceleration / weaving area). However, for freeway diverge, merge, and small weaving segments with a standard design, the length L can be considered as a constant and it can be considered implicitly in parameter a . Furthermore, in HBS 2015, only a default departing volume q_A is

considered for small weaving segments and therefore small weaving areas can be actually considered as on-ramps (entrances) as simplification. Thus, the model parameter a depends actually only on the geometrical configuration, that is, on the predefined type of merge, diverge, and small segments.

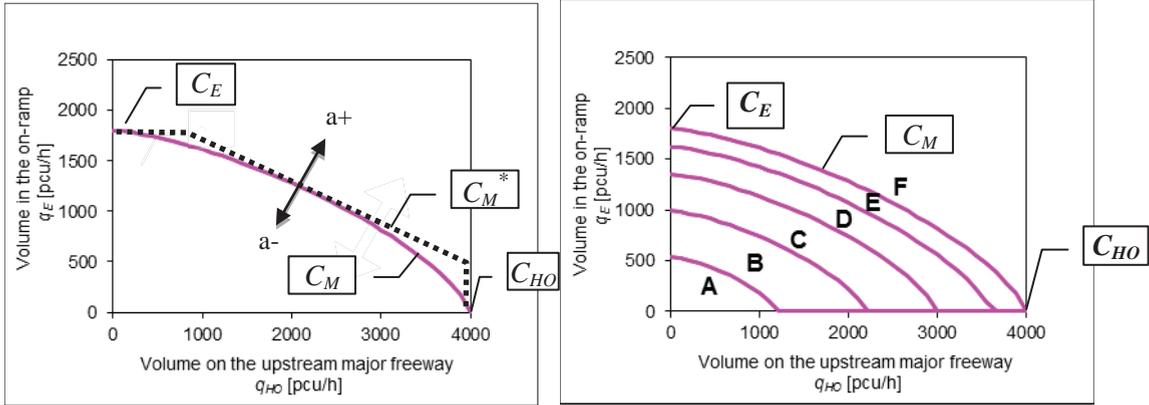


Figure 2: left: calibration of parameter a ;
right: illustration of LOS for a merge segment according to the new model

If the volume-to-capacity ratios on the major freeway (x_{HO} and x_{HU}) and on the ramps (x_E and x_A) are given, the combined volume-to-capacity ratio of the total merge or diverge (including small weaving) segment (here x_M and x_D) can be directly calculated as follows.

$$x_M = \sqrt[a]{x_E^a + x_{HO}^a} \quad \text{and} \quad x_D = \sqrt[a]{x_A^a + x_{HU}^a} \quad (5)$$

The combined LOS of the total merge or diverge segment can be then obtained according to the predefined x -value of LOS (Table 1).

If the service volume-to-capacity ratio for the merge or diverge segment is predefined ($x_{M,LOS}$ and $x_{D,LOS}$), the allowed volume-to-capacity ratios on the major freeway (x_{HO} and x_{HU}) and on the on-ramp or off-ramp (x_E and x_A) can be calculated from the following equations.

$$q_{E,LOS} = C_E \cdot x_{M,LOS} \cdot \sqrt[a]{1 - \left(\frac{q_{HO}}{C_{HO} \cdot x_{M,LOS}}\right)^a} \quad \text{and} \quad q_{A,LOS} = C_A \cdot x_{D,LOS} \cdot \sqrt[a]{1 - \left(\frac{q_{HU}}{C_{HU} \cdot x_{D,LOS}}\right)^a} \quad (6)$$

With these functions, the traffic quality (LOS) for different values of q_{HO} and q_E (or q_{HU} and q_A respectively) can be obtained directly. The right side picture in Figure 2 shows a typical shape of the new function for an on-ramp segment. It can be clearly seen, that at $q_{HO} = 0$ and at $q_E = 0$ the traffic quality of the on-ramp and the traffic quality of the major freeway are decisive for the total segment.

This model is incorporated into HBS 2015. Compared to the model in the previous HBS 2001, the new model in HBS 2015 has the following advantages: a) a uniform function for all types of freeway merge, diverge, and weaving segments, b) traffic quality assessment for three critical areas in one step (on-ramp / off-ramp, major freeway upstream / downstream, merge / diverge / weaving maneuver area), c) all boundary conditions satisfied, and d) easy calibration.

Based on the data of HBS 2001 and some recent investigations (Brilon and Betz, 2008; Weiser et al., 2006; Friedrich et al, 2006 and 2008), the new model was calibrated. Using the new model monographs for all types of merge, diverge, and weaving segments of the current German design guidelines RAA (FGSV, 2008) are constructed in HBS 2015.

2.2 Parameters Used in the Chapter in HBS 2015

In HBS 2015, parameters for all types of diverge, merge, and weaving segments of the current German design guidelines RAA (FGSV, 2008) are given for producing monographs using eq.(6). These parameters can also be used for calculating the combined volume-to-capacity ratio using eq.(5).

It is to be noted, that the unit used in this chapter is passenger car per hour (pc/h) for vehicle volume and capacity. Thus, all vehicular volumes are required to be transformed in to pc/h. For heavy vehicles, a passenger-car equivalent PCE of $E_{HV} = 2$ pc/truck is used. For upgrade loop-ramps, the value is risen to $E_{HV} = 2.5$ pc/truck.

The types of considered diverge, merge, and small weaving segments are depicted in Figure 3 and Figure 4.

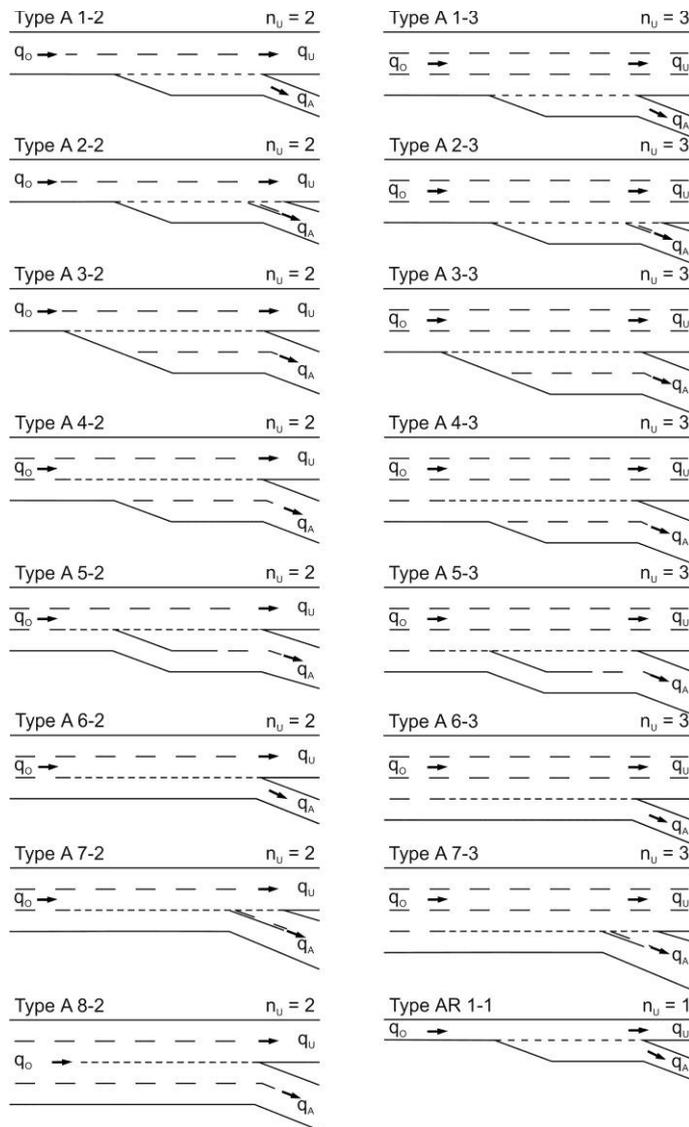


Figure 3: Diverge Segments (off-ramps) on freeways in HBS 2015 according to FGSV (2008)

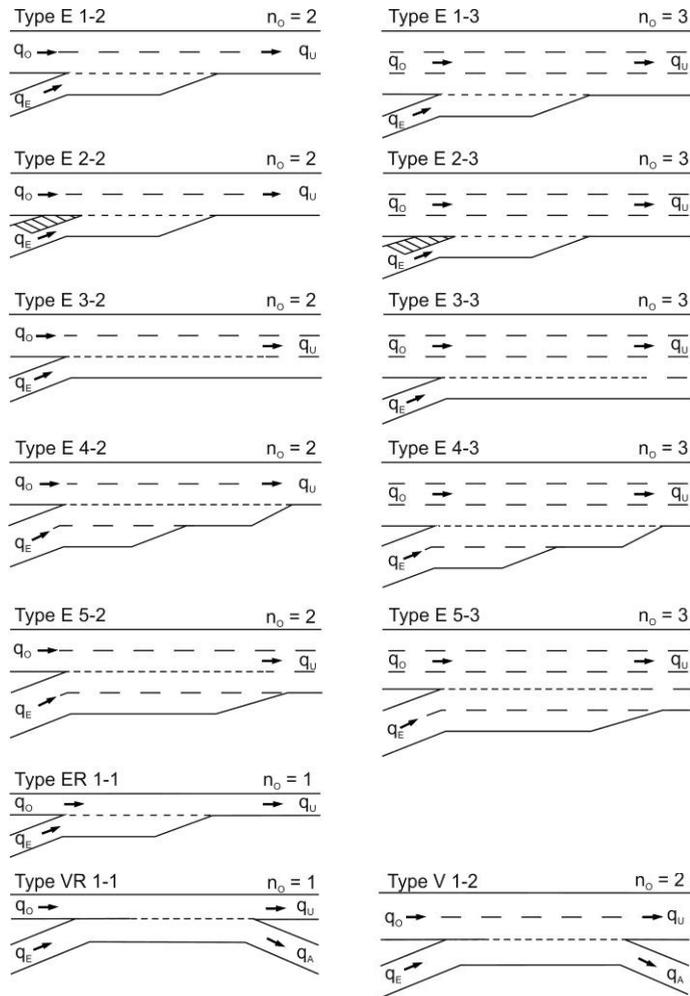


Figure 4: Merge segments (on-ramps) and small weaving segments freeways in HBS 2015 according to FGSV (2008)

The parameters used are shown in Table 2 and Table 3. Using this model of combined volume-to-capacity ratio monographs for all types of merge, diverge, and small weaving segments of the current German design guidelines RAA (FGSV, 2008) are constructed in HBS 2015. In Figure 5, two examples of those monographs are illustrated for the on-ramp type E 1-2 and the off-ramp type A 1-2.

Type	a [-]	C_A [pc/h]	C_{HU} [pc/h]
A 1-2	1,9	1800	4000
A 1-3	1,9	1800	5800
A 2-2	1,2	3060	4000
A 2-3	1,4	3060	5800
A 3-2	1,1	3600	4000
A 3-3	1,3	3600	5800
A 4-2/A 5-2	1,9	3600	4000
A 4-3/A 5-3	2,5	3600	5800
A 6-2	2,7	2000	4000
A 6-3	4,0	2000	5800
A 7-2	2,0	3060	4000
A 7-3	2,9	3060	5800
A 8-2	6,0	3600	4000
AR 1-1	1,2	1800	2000

Table 2: Parameters for diverge segments (off-ramps) in HBS 2015

Type	a [-]	C_E [pc/h]	C_{HO} or C_{VO} [pc/h]
E 1-2 / E 2-2	1,5	1800	4000
E 1-3 / E 2-3	2,1	1800	5800
E 3-2	2,7	2000	4000
E 3-3	3,8	2000	5800
E 4-2	1,05	3600	4000
E 4-3	1,3	3600	5800
E 5-2	1,8	3800	4000
E 5-3	2,4	3800	5800
ER 1-1	1,2	1800	2000
VR 1-1	1,4	1800	2000
V 1-2	1,5	1800	4000

Table 3: Parameters for merge segments (on-ramps) and small weaving segments in HBS 2015

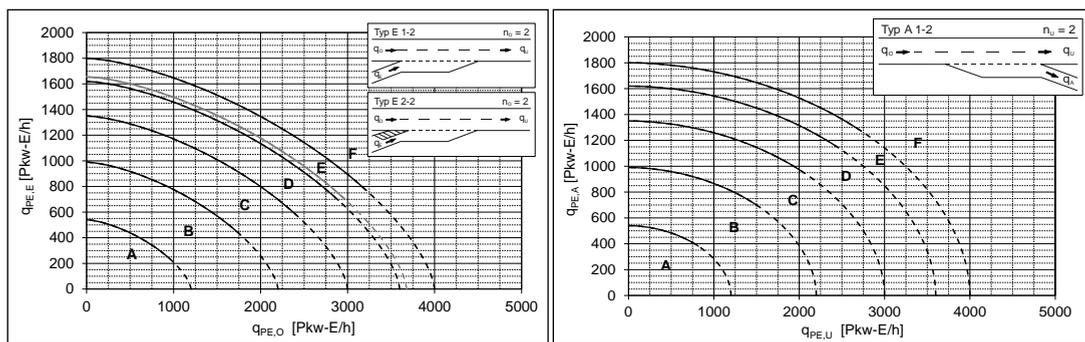


Figure 5: Examples of monographs: on-ramp type E 1-2 / E 2-2; off-ramp type A 1-2
(Translation: Typ = Type; Pkw-E, PE = pc)

Because the upstream and downstream freeways may have different capacities depending what type of freeway is actually involved and the unit of freeway capacities is different (veh/h), some of

those freeway capacities can be less than the capacities calculated in this chapter (cf. the areas with dotted line in Figure 5). Thus, the upstream or downstream freeway cross-sections have to be examined against the corresponding freeways capacities. The capacities of basic freeway segments in HBS 2015 are given in Table 4.

No. of lanes each direction	Grade s	Capacity C [veh/h]							
		Long distance freeways				Metropolitan traffic freeways			
		Percent of trucks b_{HV}				Percent of trucks b_{HV}			
		$\leq 5\%$	10%	20%	30%	$\leq 5\%$	10%	20%	30%
2	3%	3600	3500	3300	3100	3800	3700	3500	3300
	4%	3400	3300	3100	2900	3600	3500	3300	3100
	5%	3100	3000	2800	2600	3300	3200	3000	2800
3	3%	5200	5100	4800	4500	5600	5400	5100	4800
	4%	4900	4800	4500	4200	5300	5100	4800	4500
	5%	4500	4400	4100	3800	4900	4700	4400	4100
4	3%	7100	6900	6500	6100	7600	7400	6900	6400
	4%	6800	6600	6200	5800	7300	7100	6600	6100
	5%	6200	6000	5600	5200	6700	6500	6000	5500
2+ hard shoulder running	3%	4600	4500	4300	4100	5100	4900	4600	4300
	4%	4400	4300	4100	3900	4900	4700	4400	4100
	5%	4100	4000	3800	3600	4600	4400	4100	3800
3+ hard shoulder running	3%	6200	6100	5800	5500	6900	6700	6300	5900
	4%	5900	5800	5500	5200	6600	6400	6000	5600
	5%	5500	5400	5100	4800	6200	6000	5600	5200

Table 4: Capacities of basic freeway segments in HBS 2015

3 The Chapter in HBS 2015 for Diverge, Merge, and Small Weaving Segments on Rural Highways

In HBS 2015, LOS on rural highways is defined according the traffic density k . This applies also for merge, diverge and small weaving segments. The definition of LOS is given in Table 5. While for single carriageway roads (two-lane and two+one-lane roads(RQ 15,5)) the density per lane ln is considered, on short sections of dual carriageway roads (RQ 21) in the rural road network the density applies to the entire carriageway (2 lanes).

LOS	single carriageway	dual carriageway
	2 or 3 lanes	4 lanes
	Density $k_{ln,LOS}$ [veh/km/ln]	Density k_{LOS} [veh/km/ln]
A	≤ 3	≤ 9
B	≤ 6	≤ 18
C	≤ 10	≤ 30
D	≤ 15	≤ 40
E	≤ 20	≤ 48
F	> 20	> 48

Table 5: Density thresholds for defining LOS of rural highways in HBS 2015

All types of applicable merge, diverge, and small weaving segments on rural highways are illustrated in Figure 6 and Figure 7.

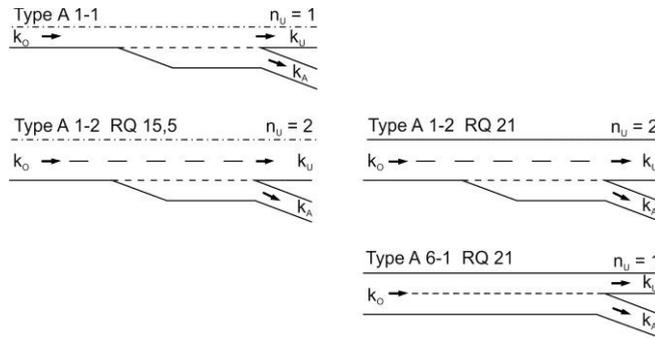


Figure 6: Diverge segments (off-ramps) on rural highways in HBS 2015
(RQ 15,5 = 2+1 cross section, RQ 21 = dual carriageway according to FGSV (2012))

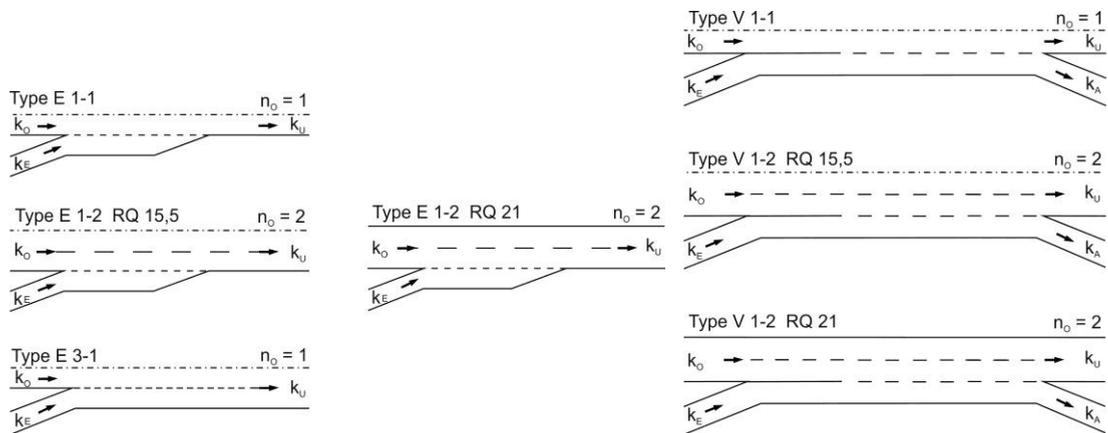


Figure 7: Merge segments (on-ramps) and small weaving segments on rural highways in HBS 2015
(RQ 15,5 = 2+1 cross section, RQ 21 = dual carriageway according to FGSV (2012))

If the density of the upstream (or downstream) major road and of the on-ramp (or off-ramp) are known, the density of the total merge (or diverge) segment is also known. In general, one of the LOS of the on-ramp (off-ramp respectively) and of the merge (diverge respectively) area is decisive. Thus, the LOS of the total merge and diverge segment can be written as follows.

$$LOS_M = \max \begin{cases} LOS_E = f(k_E) \\ LOS_M^* = f(k_E + k_O) \end{cases} \quad \text{and} \quad LOS_D = \max \begin{cases} LOS_A = f(k_A) \\ LOS_D^* = f(k_A + k_U) \end{cases} \quad (7)$$

- with k_Y = density of the corresponding cross-section Y , veh/km
 O, U = index for the upstream and downstream major road
 E, A = index for the on-ramp (entrance) and off-ramp (exit)
 M, D = index for the total merge (or weaving) and diverge segment
 M^*, D^* = index for the merge (or weaving) and diverge area

The equation for producing monographs is

$$k_{A,LOS} = \min \begin{cases} k_{U,LOS}^* - k_U \\ k_{A,LOS}^* \end{cases} \quad \text{and} \quad k_{E,LOS} = \min \begin{cases} k_{O,LOS}^* - k_O \\ k_{E,LOS}^* \end{cases} \quad (8)$$

The parameters in eq. (8) are given in Table 6 and Table 7 in combination with Table 5.

Type	Downstream major road $k_{U,LOS}^*$	Exiting stream $k_{A,LOS}^*$
A 1-1	$k_{In,LOS}$	$k_{In,LOS}$
A 1-2 RQ 15,5	$k_{In,LOS} \cdot 2$	$k_{In,LOS}$
A 1-2 RQ 21	k_{LOS}	$k_{In,LOS}$

Table 6: Density parameters for diverge segments (off-ramps) on rural highways in HBS 2015 according to Table 5

Typ	Upstream major road $k_{O,LOS}^*$	Entering stream $k_{E,LOS}^*$
E 1-1	$k_{In,LOS}$	$k_{In,LOS}$
E 1-2 RQ 15,5	$k_{In,LOS} \cdot 2$	$k_{In,LOS}$
E 1-2 RQ 21	k_{LOS}	$k_{In,LOS}$
V 1-1	$k_{In,LOS}$	$k_{In,LOS}$
V 1-2 RQ 15,5	$k_{In,LOS} \cdot 2$	$k_{In,LOS}$
V 1-2 RQ 21	k_{LOS}	$k_{In,LOS}$

Table 7: Density parameters for merge segments (on-ramps) and small weaving segments on rural highways in HBS 2015 according to Table 5

Monographs for all types of diverge, merge, and weaving segments of the current German design guidelines for rural roads RAL (FGSV, 2012) are constructed in HBS 2015. In Figure 8 two examples of those monographs are illustrated for the on-ramp types E 1-1 and E 1-2 RQ15.5.

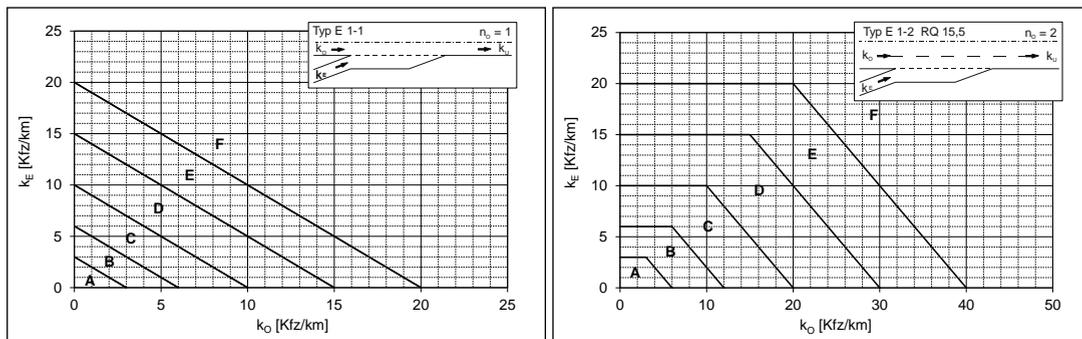


Figure 8: Examples on-ramp types E 1-1 and E 1-2 RQ15.5 on rural highways
(Translation: Typ = Type; Kfz = veh)

4 Two new Cases for Merge and Diverge Segments on 4 Lane Carriageways

In HBS 2015 no on-ramps or off-ramps are defined for freeways with four directional lanes. Fortunately, Brilon et al. (2007) also investigated the on-ramp type E 1-4 and the off-ramp type A 1-4 (see Figure 9). Thus, the parameter a for the on-ramp type E 1-4 and the off-ramp type A 1-4 can be calibrated (cf. Figure 1, left) to the measured capacities C_M^* (for E 1-4, merge area) and C_D^* (for A 1-4, diverge area) in Brilon et al. (2007). The corresponding monographs for the two additional ramp segments can be obtained (Table 1 and Figure 10) either.

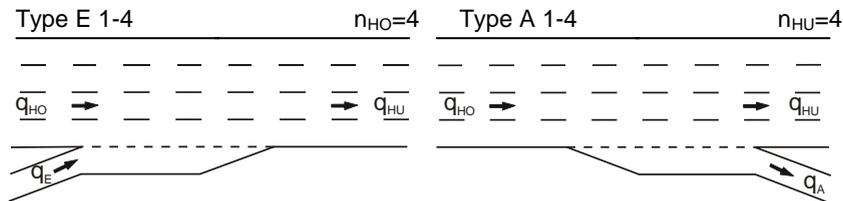


Figure 9: On-ramp type E 1-4 and off-ramp type A 1-4 on freeways

Type	a [-]	C_E or C_A [pc/h]	C_{HO} or C_{HU} [pc/h]
A 1-4	2.2	1800	8000
E 1-4 / E 2-4	2,1	1800	8000

Table 8: Parameters for on-ramp E 1-4 and off-ramp A 1-4 on freeways

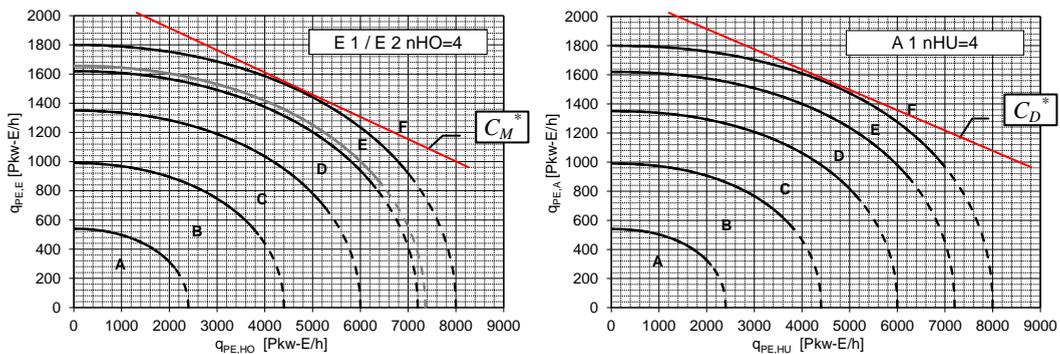


Figure 10: Monographs for on-ramp E 1-4 and off-ramp A 1-4 on freeways (Translation: Pkw-E, PE = pc)

The results of the on-ramp type E 1-4 applies also to the on-ramp type E 2-4.

5 Summary and Outlook

In the new German Highway Capacity Manual (HBS) (FGSV, 2015), the chapters for diverge, merge, and small weaving segments is rewritten in accordance with forthcoming developments in the past 10 years. In this paper, an overview of the chapters for diverge, merge, and small weaving segments on freeways and rural highways in HBS 2015 is presented. Procedures dealing with performance analyses and level of service (LOS) of those segments are introduced both for freeways

and rural highways. Differences between the former version and the new version of the chapter in HBS are indicated and discussed. In addition, two additional cases for merge and diverge segments on 4 lanes carriageways which are not yet included in HBS 2015 are investigated according to available data. For the two new cases, calibrated parameters and monographs are provided for the further development of HBS.

References

- FGSV (2001). *Handbuch für die Bemessung von Straßenverkehrsanlagen (HBS) (German Highway Capacity Manual)*. Forschungsgesellschaft für Straßen- und Verkehrswesen (Ed.), Nr. 299, FGSV Verlag GmbH, Cologne.
- FGSV (2008), *Richtlinien für die Anlage von Autobahnen (RAA) (Guidelines on the Design of Motorways)*. Forschungsgesellschaft für Straßen- und Verkehrswesen, Cologne, 2008, translation 2011.
- FGSV (2012), *Richtlinien für die Anlage von Landstraßen (RAL) (Guidelines on the Design of Rural Roads)*. Forschungsgesellschaft für Straßen- und Verkehrswesen, Cologne, 2012.
- FGSV (2015). *Handbuch für die Bemessung von Straßenverkehrsanlagen (HBS2015) (German Highway Capacity Manual)*. Forschungsgesellschaft für Straßen- und Verkehrswesen (Ed.), Nr. 299, FGSV Verlag GmbH, Cologne.
- TRB (2000). *Highway Capacity Manual (HCM2000)*. TRB, Transportation Research Board, National Research Council, Washington, D.C.
- TRB (2010). *Highway Capacity Manual (HCM2010)*. TRB, Transportation Research Board of the National Academies, Washington, D.C.
- Brilon, W., Westphal, S. (1994). Bemessung von Einfahrten an planfreien Knotenpunkten (Traffic quality of merge segments). *Forschung Straßenbau und Straßenverkehrstechnik*, No. 671.
- Brilon, W., Betz, C. (2008). Entwurf und Bemessung von Autobahnknotenpunkten unter Berücksichtigung der Wechselwirkungen zwischen den Elementen (Design and evaluation of motorway intersections, considering the interactions between elements). *Forschung Straßenbau und Straßenverkehrstechnik*, No. 983
- Brilon, W.; Geistefeldt, J.; Lippold, C.; Kuczora, V. (2007): Autobahnen und Autobahnknotenpunkte mit vierstreifigen Richtungsfahrbahnen – Gestaltung und Bemessung (Planning and design of motorway sections and interchanges with four-lane carriageways), *Forschung Straßenbau und Straßenverkehrstechnik*, No. 967.
- Friedrich, B., Irzik, M., Hoffmann, S. (2006). Verkehrsablauf an Autobahnausfahrten des Typs A 2 der RAL-K-2 (Traffic quality of diverge segment typ A2). *Forschung Straßenbau und Straßenverkehrstechnik*, No. 946.
- Friedrich, B., Hoffmann, S., Irzik, M., Matschke, I. (2008). Zweistreifige Einfahrten an dreistreifigen Richtungsfahrbahnen – Verkehrssicherheit, Verkehrsablauf (Two-lane on-ramps on three-lane freeways). *Forschung Straßenbau und Straßenverkehrstechnik*, No. 992.
- Weiser, F., Sillus, A., Lippold, C. (2006). Verkehrssicherheit und Verkehrsablauf an planfreien Knotenpunkten mit Fahrstreifensubtraktion (Traffic quality of diverge segments with lane reduction on major freeway). *Forschung Straßenbau und Straßenverkehrstechnik*, No. 936.
- Wu, N. (1997). Capacity of Shared-short Lanes at Unsignalized Intersections. *Proceedings of the Third International Symposium on Intersections without Traffic Signals*, Portland, Oregon
- Wu, N., Lemke K. (2011). A new Model for Level of Service of Freeway Merge, Diverge, and Weaving Segments. *Procedia Social and Behavioral Sciences, 6th International Symposium on Highway Capacity and Quality of Service*. Volume 16, Elsevier, Stockholm, Sweden, June 28 – July 1.